JOURNAL OF AIRCRAFT
Vol. 45, No. 4, July–August 2008

Effect of Jerk and Acceleration on the Perception of Motion Strength

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DOI: 10.2514/1.33757

In a flight simulator, the calculated aircraft motions are scaled down and filtered to fit within the envelope of the simulator motion system. A number of recent flight and ground simulation studies have reported that the simulator motion was too strong, when in fact, the motion was scaled down and filtered. This paper puts forth the hypothesis that this could be due in part to the motion drive algorithm and vehicle model exaggerating the jerk. To test the plausibility of this hypothesis a paired-comparison experiment was run to determine if the subjective impression of motion strength is a function of both the acceleration and jerk of the motion. The experiment found that the level of jerk and acceleration contributed to the perceived strength of motion, with larger jerks and accelerations leading to increased motion strength. In addition, the duration of the acceleration had a significant effect on the perceived motion strength, with longer durations leading to increased motion strength. Although the relationship between jerk and motion strength suggests that exaggerated jerk in the simulator could lead to the preference for scale factors less than one, the strength of the relationship strongly suggests that it does not entirely account for the preference.

Nomenclature

A_i = nominal acceleration of motion condition i, m/s^2
A_{nom} = acceleration used to normalize the linear model, m/s^2
a_i = total score of motion condition i (number of times selected)
D_i = length of constant-acceleration phase of motion, s
D_n = score variable for model testing
J_i = nominal jerk level of motion condition i, m/s^3
J_{nom} = jerk level used to normalize the linear model, m/s^3
l = actuator length command, m
n = total number of comparisons between two conditions
p = probability of null hypothesis
S_x = simulator fore–aft displacement command, m
s = Laplace variable
T_J = duration of the constant jerk section of the motion condition, s
T_s = duration of half of the motion condition, s
t = number of motion conditions
V_x = motion-based fore–aft velocity, m/s
$\ddot{x}_c$ = commanded X acceleration
$\ddot{x}_i$ = intermediate X
α_{ij} = total number of times that condition i was preferred over condition j
β_A = perceived motion strength change due to a change in acceleration
β_A^* = normalized perceived motion strength change due to a change in acceleration
β_D = perceived motion strength change due to a change in the duration of constant acceleration
β_i = perceived motion strength of motion condition i, no order effect
β_i^* = perceived motion strength of motion condition i, order effect
β_J = perceived motion strength change due to a change in jerk
β_J^* = normalized perceived motion strength change due to a change in jerk
γ = model order effect
θ = simulator pitch command, rad
π_i = preference probability for condition i
π_{ij} = probability that condition i is selected over condition j, no order effect
π_{ij}^* = probability that condition i is selected over condition j, when i is the first condition
σ = standard deviation
χ^2 = χ^2 distribution
$\dddot{\omega}$ = angular jerk, time derivative of angular acceleration
$\hat{}$ = estimated parameter based on maximum likelihood
$\ddot{}$ = second time derivative

Presented as Paper 6253 at the AIAA Modeling Simulation Technologies Conference and Exhibit, Keystone, CO, 21–24 August 2006; received 10 August 2007; accepted for publication 22 November 2007.

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I. Introduction

PILOTS close the control loop around the aircraft using a number of different cues. Simulators attempt to provide as many of these cues as possible, with varying degrees of fidelity. The intent is to provide the pilot with the required information regarding the state of the aircraft such that they can control it using the same techniques that they use to control the real aircraft. In addition, high-fidelity cues add to the face validity of the simulator. The simulator motion system is used to stimulate the human's vestibular, proprioceptive, and somatosensory systems, which are thought to provide high-frequency lead information that can be important for controlling unstable or marginally stable vehicles, particularly when random disturbances are present [1–3].

A number of recent flight and driving simulation studies have found a somewhat surprising result: subjects have reported that the simulator motion was too strong when, in fact, the calculated vehicle motions were scaled down and filtered [4–7]. In an experiment by Groen et al. [4], pilots rode as passive observers in a simulator during playback of a simulated takeoff run. The scale factor applied to the specific forces was varied from 0 to 1.0, and the break frequency of the high-pass (washout) filter was adjusted to achieve a constant surge displacement of 1.3 m. In addition, the tilt-coordination scale factor was also varied from 0 to 1.0, and the low-pass tilt-coordination filter break frequency was kept constant. Pilots were asked to rate the motion as being either too weak or too powerful. For the initial onset of motion, it was found that a surge gain of 0.2 and a tilt-coordination gain of 0.6 was preferred and that there was unanimous agreement that a gain of one produced motion that was too strong. According to Groen et al., the difference in the high-pass-filter break frequencies may have confounded the results. The authors go on to suggest that scaling of motion may be required, due to the tendency of subjects to overestimate physical motion relative to visual motion in virtual environments. In other words, the authors hypothesize that the perception of exaggerated simulator motion results from a larger perceptual scale factor for the motion cues compared with the perceptual scale factor for the visual cues.

Although certainly possible, there may be a simpler explanation for why scale factors of less than one led to motion that was perceived as too strong. It is thought that humans are not only sensitive to specific force, but may also be sensitive to the time derivative of specific force [8]. We will refer to this time derivative, somewhat erroneously, as jerk (the jerk is really just the time derivative of the acceleration, not specific force). The jerk level can often be larger in the simulator than in the real vehicle, even when the specific-force levels are substantially smaller. The main contributors to this jerk distortion in the simulator are the vehicle dynamics model and the motion drive algorithm (MDA). The vehicle models are often relatively simple, to make the equations tractable for real-time simulation. Many of the simplifications that are made can lead to increased stiffness of the vehicle dynamics, which in turn can lead to higher jerk levels. A simple example for aircraft simulation is the assumption of quasi-steady aerodynamics. The actual change of aerodynamic forces for a change in the aerodynamics states does not change as quickly as predicted by quasi-steady aerodynamics. This leads to more rapid changes in acceleration and hence higher levels of jerk. Similarly, in ground vehicle simulators, the tire carcass torsional dynamics and suspension bushings are often ignored, which can cause excessive jerk in the simulator. In the experiment by Groen et al. [4], a step input of 3.5 m/s^2 in longitudinal specific force was used to simulate the initial takeoff roll of an aircraft. The jerk associated with the step input is equal to the size of the step in acceleration divided by the update rate of the simulation (which is usually at least 60 Hz, leading to a jerk of at least 210 m/s^3 in this case). This is unrealistically large: even for a full-throttle brake release, the time required for the brakes to go from fully depressed to fully released limits the jerks to approximately 30 m/s^3.

The high-pass filters in the MDA [9] can also lead to extraneous jerk cues in the simulator that may be interpreted as exaggerated motion. The response of a second-order high-pass filter to a band-limited step input in specific force is shown in Fig. 1 (case HP). The step input was band-limited with a 40-dB/decade roll-off above 30 rad/s. The high-pass filter had a break frequency of 2.5 rad/s and a damping of 1. As seen in the figure, the specific force and jerk match the step input initially, but shortly after, the step onset the acceleration decays; therefore, there is an additional jerk cue (in the opposite direction) in the simulator, associated with the reduction of the acceleration. The peak-to-peak jerk level is therefore slightly amplified in this case, even though the acceleration is somewhat reduced. The high-pass-filter parameters used are typical simulator values, similar to the CW2 parameter set described by Reid and Nahon [10].

A potentially larger contributing factor to the distortion of jerk is the angular limiting that is often applied in the low-pass tilt-coordination circuit of the MDA [9]. Second-order low-pass filters with internal rate limiters are often employed in the tilt-coordination circuit. This can lead to a very large angular jerk $\ddot{\omega}$ when the rate limiter becomes active [11]. This in turn leads to a very large translational jerk, even when the center of rotation is close to the pilot. The response of a high-pass/low-pass (with rate limiter) combination at a point 1.78 m above the center of rotation is shown in Fig. 1 (case HP+LP). The filter parameters used to generate this were the CW2 coefficients described by Reid and Nahon [10], except that the low-pass break frequency was reduced from 5 to 1.8 rad/s to be more representative of current tuning strategies, and the scale factor was taken to be unity. In this case, the maximum peak-to-peak jerk in the simulator (due to the tilt-rate limiter) is approximately four times the value in the aircraft. The onset specific force at the pilot location has a gain much smaller than 1, due to the specific-force cue caused by the angular acceleration from the tilt-coordination acting through a radius of 1.78 m.

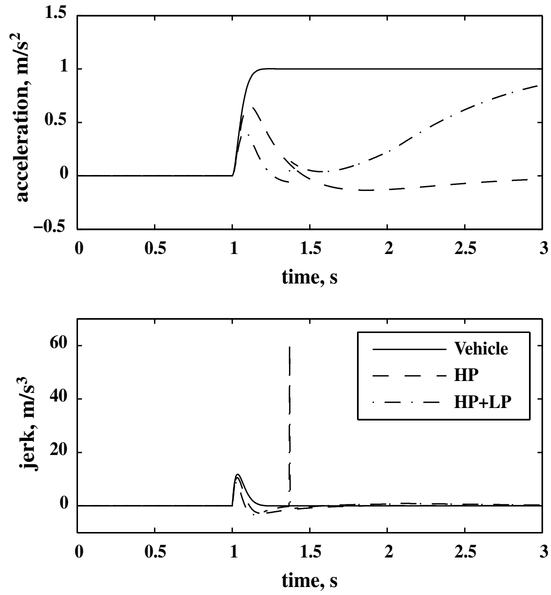


Fig. 1 MDA effect on jerk and acceleration.

Although this demonstrates that the jerk may be exaggerated in the simulator, the effect of this on perceived motion strength cannot be predicted from the literature. Hosman [8] argued that the types I and II hairs within the otolith not only respond to deflection (which measures specific force) but also to the deflection rate (which measures jerk). However, in simulator motion threshold experiments, Hosman and van der Vaart [12] found that the threshold for sinusoidal fore–aft motion was not significantly effected by frequency. A more recent experiment by Heerspink et al. [13] examined a larger range of frequencies than did Hosman and van der Vaart [12] and found that higher frequencies had lower thresholds (supporting the claim of jerk sensitivity). A previous study by Mah et al. [14] found contradictory results. Thresholds were determined over a range of frequencies from 0.2 to 3 Hz. At low frequencies (0.2 to 0.5 Hz), the thresholds were found to be independent of frequency, indicating no dependence on jerk. At higher frequencies, the thresholds increased with frequency, indicating some kind of integrating behavior (as opposed to a differentiating behavior that would be expected if jerk was important). Mah et al. used a high-performance longitudinal sled that produced very smooth motion. Effort was also taken to ensure that the subject received minimal cues from sources other than their vestibular system. The subjects' heads were restrained and pressure-relieving foam was used on the seat and belts to reduce the somatosensory cues to the subject. Although this is obviously necessary to test for vestibular-system-based jerk perception, it eliminated a number of potential jerk-sensing cues that are available to subjects in a flight or ground simulator. Jerk tends to push the subject in and out of their seat and provides changing forces on the head that must be balanced by the neck muscles. The reduction of these cues in the Mah et al. experiment could explain the discrepancy between these and the Heerspink et al. [13] threshold results. Alternatively, Mah et al. [14] argued that extraneous cues (i.e., nonsmooth motion) may have caused the apparent jerk effect in previous threshold experiments. An experiment by Greig [15] provided data that tend to contradict this hypothesis. Greig found that extraneous noise did not affect the motion thresholds unless the bandwidth of the noise overlapped with the signal. In the Heerspink et al. [13] experiment, it is unlikely there was significant power in the noise at the relatively low driven frequencies, because the simulator, motion, and navigation (SIMONA) simulator is a high-performance state-of-the-art simulator. Furthermore, it is difficult to extend threshold experiments to the suprathreshold motions that are of interest in this study. It is unknown if the effect of jerk at low levels of both acceleration and jerk will carry over to higher acceleration and jerk levels that occur during typical flight training.

In a recent study, Huang and Wang [16] examined a large number of ground vehicle transmission shifts and found that jerk and acceleration can have a significant impact on vehicle shift harshness, with increased jerk leading to increased harshness. This appears in contrast with an earlier study by Duncan and Wegscheid [17], which found that for tractor shift quality, acceleration was much more important than jerk and that reduced jerk was actually associated with reduced shift quality. It seems that in Duncan and Wegscheid's study, the experienced tractor drivers associated high jerk levels with crisp shifts and therefore higher jerks were rated as superior. In addition, the hydraulic motion system used to create the shift time histories in Duncan and Wegscheid's study appears to have significantly distorted the desired shift histories. There is thus contradictory evidence on how the combination of jerk and acceleration contributes to shift quality and the strength-of-motion perception. An experimental study was therefore designed to answer this question.

II. Experimental Method

The paired-comparison methodology [18] was used to compare nine different fore–aft motion time histories with varying levels of jerk and acceleration. The paired-comparison methodology is favored over other experimental designs, because humans can make very consistent and repeatable judgments when comparing two stimuli that are presented with a short interstimulus interval. The ideal motions were jerk-limited acceleration square waves, as shown in Fig. 2. Nine different motions were formed by the combination of three acceleration and three jerk levels, as shown in Table 1. The time T_s, shown in Fig. 2, was kept constant at 1.45 s for eight of the nine motion conditions to prevent the subjects using the duration of the maneuver to aid with the detection task. This meant that the duration of the constant-acceleration phase of the motion varied between the nine conditions. If instead of time, some other aspect of the motion had been kept constant (such as the displacement or the duration of the constant-acceleration phase) across the different acceleration/jerk levels, then the duration of the motion would have provided the subject with a cue regarding the level of the acceleration and jerk. It should be noted that T_s for motion condition I was slightly shorter than 1.45 s to keep the motion commands within the simulator displacement envelope. The maximum commanded displacement for all the maneuvers was approximately 1 m. The nine different motions were compared pairwise twice (with the order reversed for the second pair), for a total of 72 comparisons.

III. Experimental Setup

The experiment was performed in the University of Toronto Institute for Aerospace Studies (UTIAS) Flight Research Simulator (FRS), shown in Fig. 3. The FRS employs a CAE 300 series high-performance hydraulic motion system with a maximum fore–aft displacement of 1.2 m, a maximum fore–aft velocity of 0.8 m/s, and a maximum fore–aft acceleration of 10.0 m/s^2. For modest inputs, the system response is smooth and linear, with a relatively flat frequency response to approximately 10 Hz. For complete details of the FRS motion characteristics, see Grant [19]. For the relatively severe motions required for this study, the FRS motion system exhibited significant nonlinearities. These motion errors were sufficiently large that corrections were deemed necessary. The motion system's response was corrected by distorting the commanded input such that the response to this distorted input closely matched the desired motion. A trial-and-error method was used to determine the required distortion, which is shown in Fig. 4 and described next:

1) The input was run through the second-order lead-lag network given by

$$\frac{(s + 2.355)(s + 0.314)}{(s + 1.767)(s + 0.4185)} \tag{1}$$

This linear network compensates for a 2 dB dip at 0.8 rad/s in the linear frequency response of the FRS motion system (see Grant [19]).

2) A pitch command proportional to the commanded fore–aft velocity of $0.6V_x$ (degrees), was introduced. This was used to cancel out a nonlinear, unintended, pitch that was related to the velocity of the motion system. As the velocity of an actuator increases, its position-following accuracy deteriorates and the actuator lags behind the desired position. Because the fore–aft maneuver creates different velocities on the three actuators on each side of the motion system, the different position errors of the actuators lead to an unintended pitch motion. The unintended pitch creates an erroneous contribution to the x specific force from the x component of the gravity vector. The specific force from the pitch is indistinguishable from linear x acceleration, and so it needed to be removed. The calculated pitch was run through a 25-rad/s first-order low-pass filter to delay the command so that it coincided with the measured pitch of the motion system.

Table 1 Motion conditions

Motion condition	Condition index	A, m/s^2	T_J s	T_s s	J, m/s^3
A	1	0.4	0.4	1.45	1.0
B	2	0.4	0.2	1.45	2.0
C	3	0.4	0.133	1.45	3.0
D	4	0.5	0.5	1.45	1.0
E	5	0.5	0.25	1.45	2.0
F	6	0.5	0.167	1.45	3.0
G	7	0.62	0.62	1.45	1.0
H	8	0.62	0.31	1.45	2.0
I	9	0.62	0.207	1.383	3.0

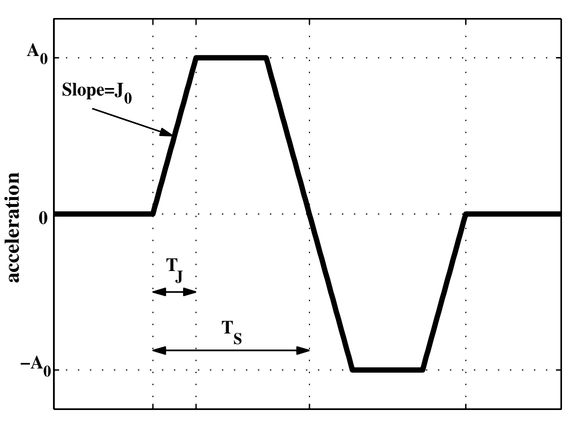


Fig. 2 Ideal, jerk-limited, fore–aft acceleration square wave.

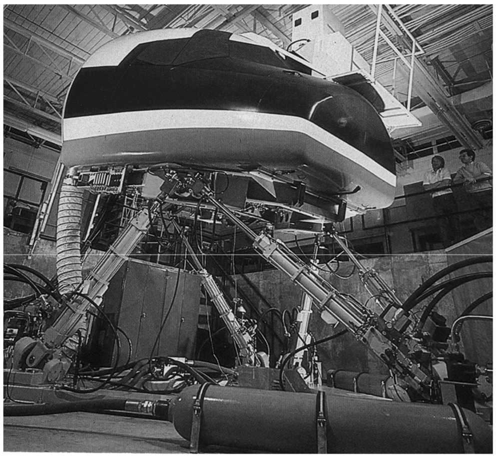


Fig. 3 The UTIAS flight research simulator.

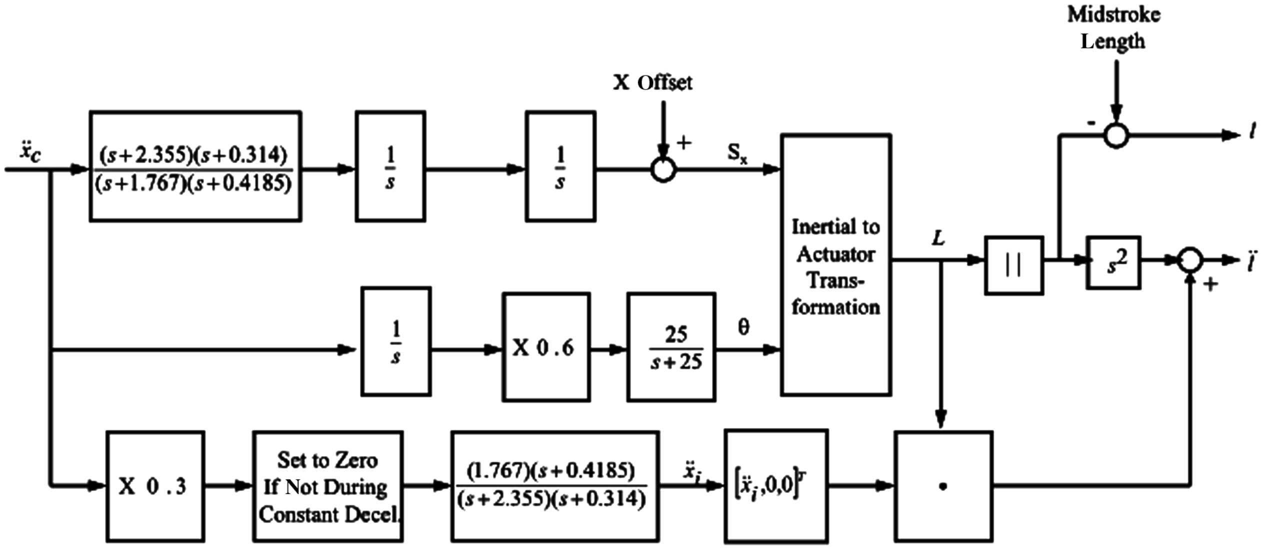


Fig. 4 Distortions of the commanded motions.

3) During the constant-deceleration phase of the input, the motion system gradually built up acceleration. In an attempt to compensate for this, an additional acceleration equal to 30% of the command was passed through a shaping filter and added onto the actuator accelerations during the constant-deceleration phase. The FRS motion system requires actuator position and acceleration command signals. This correction was only applied to the actuator acceleration commands (unlike the previous corrections, which were made to both actuator position and acceleration commands). This correction was only partially successful.

The results of these corrections are shown in Fig. 5. In this figure, the ideal specific-force command, the uncorrected specific-force response, and the corrected specific-force response (response to the distorted input) are shown for all nine motion conditions. The motion system's specific-force response was measured using a BEI MotionPak inertial measurement package. The maximum pitch angle of the simulator, after the correction was applied, was less than 0.3 deg for all motion conditions. For motion conditions A to G, the maximum lateral and vertical specific forces were less than 5 mg and the maximum roll and yaw angles were less than 0.3 deg. For motion conditions H and I, the maximum lateral and vertical specific forces were less than 8 mg and the maximum roll and yaw angles were less than 0.5 deg. For all maneuvers, the angular rates were always less than 0.5 deg/s.

An experimental trial consisted of two motion conditions. A single tone indicated to the subject that the first of the motion conditions was about to begin and two tones indicated the beginning of the second motion condition. An LCD touchscreen display also showed when the first motion condition or the second motion condition was in progress by changing the color of the appropriate numbered button to green (see Fig. 6). Before the beginning of each motion condition, the simulator was prepositioned 0.5 m rearward. This prepositioning was done using a jerk-limited acceleration square wave with a jerk level of 0.25 m/s^3 and an acceleration of 0.08 m/s^2. The jerk and acceleration levels of the prepositioning motion was a tradeoff between quick repositioning, resulting in a shorter interstimulus interval (leading to improved discrimination), and the distraction caused by the detection of the interstimulus (prepositioning) motion. The selected jerk and acceleration resulted in a motion that was just at the threshold of detection, yet only required 14 s to preposition the simulator after motion condition I (worst case). The time interval between the two motion conditions was kept constant at 14 s for all combinations of motion conditions by adding an appropriate-length quiescent interval. This was done so that the repositioning time interval would not provide the subject with a cue regarding the distance the simulator moved on the previous trial. After the second motion condition in each trial and during the prepositioning for the next trial, the subject indicated which of the two motions (first or second) was stronger. The subject was prompted for their response by audio tones and a message on the LCD display. Subjects entered their selection on the LCD touchscreen. A text message on the LCD display was also used to inform the subjects when the simulator was prepositioning. Subjects wore sealed headphones during the entire experiment, and broadband audio noise was fed into the headphones during the entire experiment to mask the actuator noise.

Twelve subjects, five female and seven male, participated in the experiment. The subjects ranged from 19 to 54 years old, with a mean of 34. The subjects were a combination of engineering students and nontechnical administrative personnel. Each subject was exposed to 72 pairs, the 36 different combinations of the nine motion conditions in the two different orders. The 72 pairs were run in two different 20-min sessions to reduce subject fatigue and boredom. The two sessions were separated by at least 3 h and, most often, by 24 h. The order of presentation of the 72 conditions was randomized for each subject.

IV. Results and Discussion

A. Nonparametric Analysis

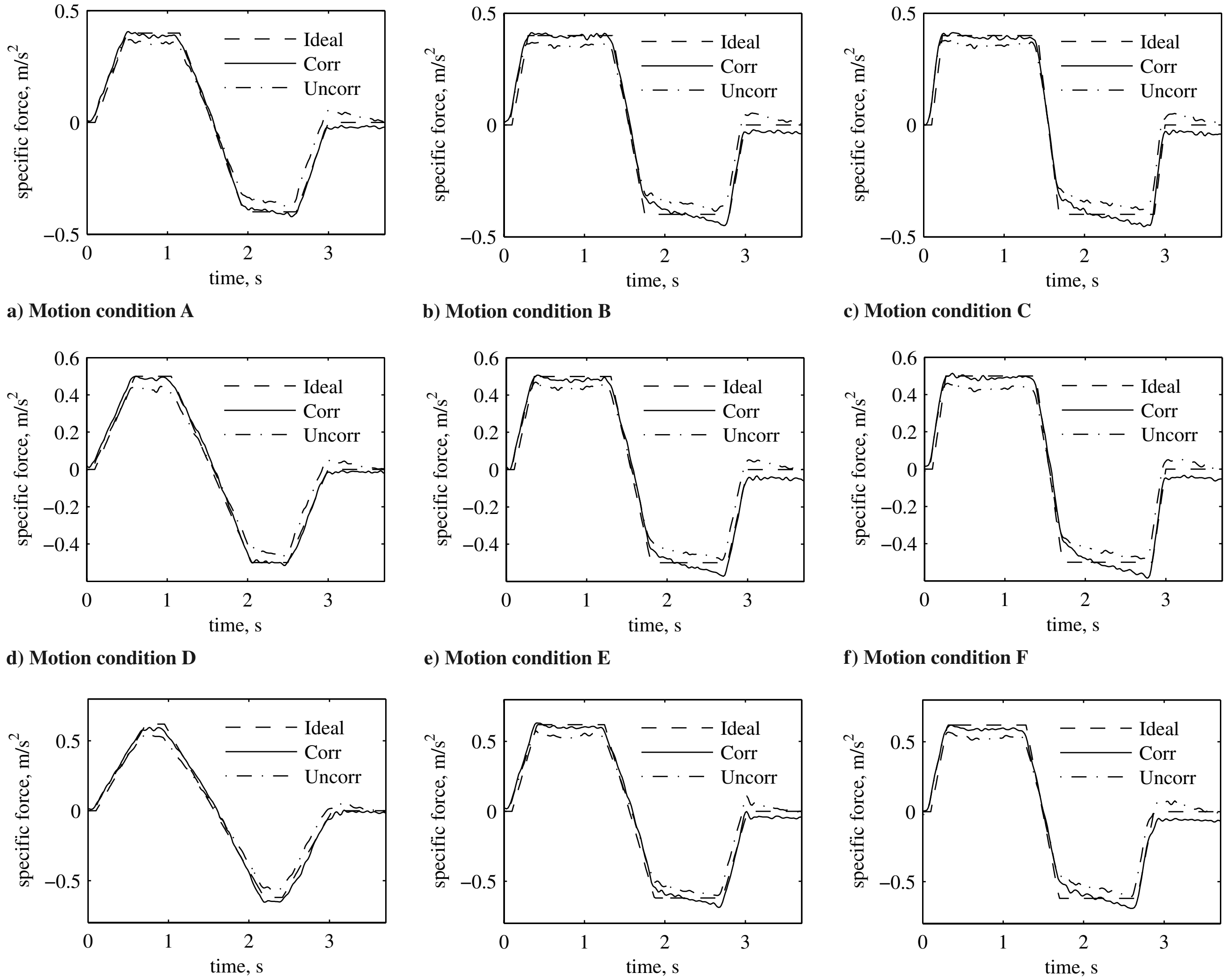
The following nonparametric analysis is based on the work of David [18]. In this analysis, there are no assumptions regarding the relationship between motion strength and paired probabilities. The probability that motion condition i is rated stronger than motion condition j is given by the pair probability π_{ij}, which is estimated using

$$\hat{\pi}_{ij} = \alpha_{ij}/n \tag{2}$$

where α_{ij} is the total number of times that condition i is selected over condition j, and n is the total number of times that i and j are compared. In the following analysis, only data from 11 subjects were analyzed, because one of the female subject's data file was corrupted. For this experiment, $n = 22$ (11 subjects times 2 orders). The total scores for each motion condition are then

$$a_i = \sum_{j=1}^{t} \alpha_{ij} \tag{3}$$

where t is the number of conditions being compared, which is 9, and α_{ii} is 0 by definition. If all the motion conditions had the same strength, then each condition would have an equal probability of $\frac{1}{2}$ during any comparison. The expected value of all the scores, when all the motion conditions have the same strength, is therefore 88 $[1/2 \times (9-1) \times 2 \times 11]$. The total scores for motion conditions A, B, C, D, E, F, G, H, and I are 22, 56, 95, 43, 99, 118, 74, 140, and 145, respectively, and these are shown in Fig. 7. The total scores plotted



a) Motion condition A
b) Motion condition B
c) Motion condition C
d) Motion condition D
e) Motion condition E
f) Motion condition F
g) Motion condition G
h) Motion condition H
i) Motion condition I

Fig. 5 Ideal, uncorrected, and corrected time histories for all motion conditions.

against acceleration and jerk are shown in Fig. 8. As seen in this figure, both acceleration and jerk seem to have a positive effect on motion strength.

The first step in the analysis is to test the null hypothesis, which is the equivalent to each motion condition having the same motion strength ($\pi_{ij} = 0.5$). David [18] showed that for the null hypothesis, the distribution of D_n,

$$D_n = \frac{4[\sum_{i=1}^{t} a_i^2 - \frac{1}{4} t n^2 (t-1)^2]}{nt} \tag{4}$$

can be approximated by the χ^2 distribution with $(t-1)$ degrees of freedom. For the given scores, the null hypothesis is rejected ($p < 0.0001$) and therefore significant differences exist between at least some of the motion conditions. The least-significant-difference test, which is analogous to carrying out paired t tests, is therefore

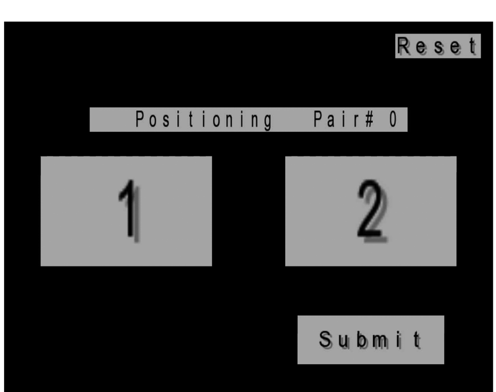


Fig. 6 The LCD touchscreen display.

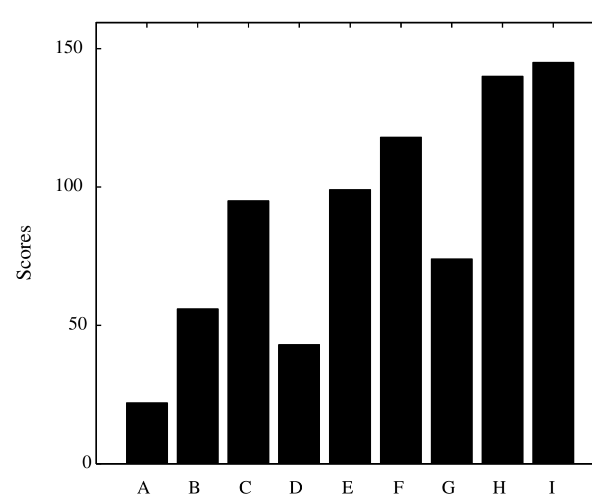


Fig. 7 Total scores for motion conditions.

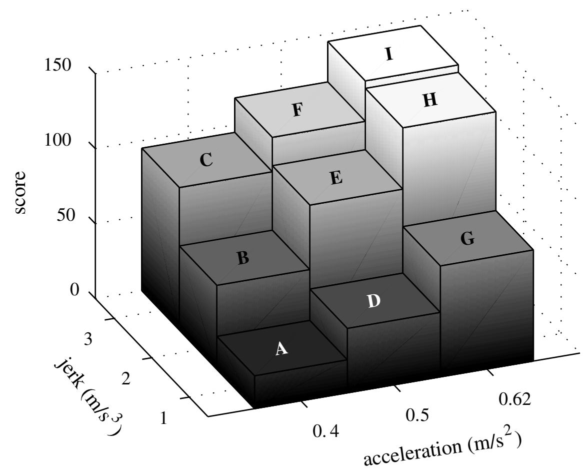


Fig. 8 Total scores versus jerk and acceleration.

A D B G C E F H I

Fig. 9 Motion condition ordering.

used to determine which of the motion conditions are significantly different from each other. For reasonably large $n(t-1)$, the distribution of the difference in motion scores will be approximately normal, with a variance given by [18]

$$\sigma = \sqrt{1/2nt} \tag{5}$$

Therefore, the critical difference in scores at the 5.6% level (5.6% was selected to get an integer difference) is 19, which results in the ordering of motion conditions shown in Figure 9. Any two motion conditions for which the scores are not underlined by the same line are significantly different at the 5.6% level. The order is also shown in Fig. 10, in which conditions that are not significantly different are enclosed within a shape, and the magnitude of the score is indicated by the shade of the shape. Thus, the lowest (significantly different) motion strengths were associated with the lowest jerk level, regardless of the acceleration. For the higher jerk levels, the ordering is not so clear.

The subject consistency can be calculated by considering the number of circular triads. A circular triad exists when any three items being compared (by a single subject) are drawn on a directed graph and the arrows point in a circular motion (with the arrows going from the preferred object to the nonpreferred object). For example, the following triad is circular: A is preferred to B, B is preferred to C, and C is preferred to A. The coefficient of consistency is defined as 1 when there are no circular triads and 0 when there are a maximum number of circular triads. The overall coefficient of consistency is the average consistency of all subjects, which is 0.65 for the current

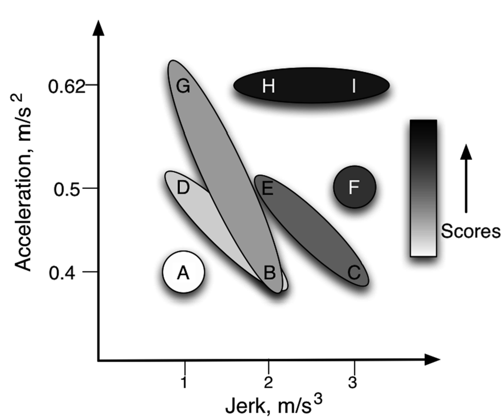


Fig. 10 Motion score ordering.

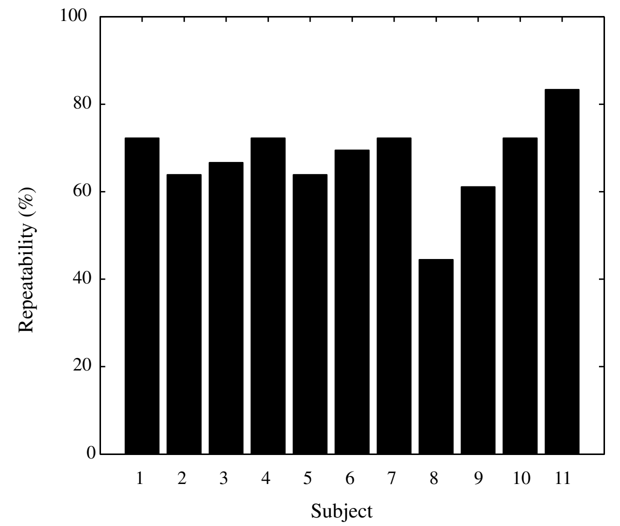


Fig. 11 Subject repeatability.

experiment. This is rather low, which could be an indication that the items are not being compared on a single scale, or it could simply indicate the conditions being compared are quite similar. This will be addressed in a later section.

Also of interest is the repeatability of each subject between the two different orders for each pair. For example, A was compared with B twice, once with A first and once with B first. The repeatability is found by comparing which condition is selected from the two reversed trials and summing over all the comparisons and dividing by the total number of comparisons. The subject repeatability is shown in Fig. 11. Subject 8 had a repeatability of less than 50%, and the average repeatability of all subjects was 67%. This relatively low repeatability could be an indication that some order effect existed. For example, the subjects may have a tendency to select the second motion condition if the two conditions were difficult to distinguish. This will be addressed in a later section.

Of particular interest for this experiment is the fact that at a given level of acceleration, the jerk level can significantly affect the score, and even more important, there are four cases in which a larger score is associated with a motion condition with a lower acceleration level but higher jerk level (see Fig. 8). For example, a jerk level of 1 m/s^3 with an acceleration level of 0.62 m/s^2 (case G) scores significantly lower than a jerk level of 3 m/s^3 with an acceleration level of 0.4 m/s^2 (case C). One must not draw too strong a conclusion from this ordering, however, because the length of the duration of the constant phase of acceleration varied systematically as a function of jerk and acceleration and, as will be shown subsequently, this had a significant affect on the motion strength.

B. Parametric Analysis

In an attempt to quantify the effect of the jerk and acceleration, a Bradley–Terry model [18] was fit to the data using a maximum-likelihood approach. The Bradley–Terry model assumes that the probability of condition i being preferred to condition j, π_{ij}, can be found from

$$\pi_{ij} = \frac{\pi_i}{\pi_i + \pi_j} \tag{6}$$

where π_i is a preference parameter, and the subjective merit of condition i, β_i, is taken to be

$$\beta_i = \log(\pi_i) \tag{7}$$

In this case, the subjective merit is the motion strength. These equations imply that the difference in motion strengths between pairs of conditions is distributed logistically. We can fit the Bradley–Terry model to the data to get an estimate of β_i, denoted as $\hat{\beta}_i$. For the current experiment, β_i is the perceived motion strength for condition

Table 2 Bradley–Terry model motion strength values

Condition	A	J	$\hat{\beta}$	$\hat{\beta}^*$
A	0.4	1	−2.03	−2.24
D	0.5	1	−1.26	−1.39
B	0.4	2	−0.86	−0.95
G	0.62	1	−0.36	−0.40
C	0.4	3	0.20	0.22
E	0.5	2	0.31	0.34
F	0.5	3	0.83	0.91
H	0.62	2	1.5	1.66
I	0.62	3	1.68	1.85

i. It should be noted that the subjective motion strength has an arbitrary zero point; only the difference in motion strengths is meaningful. The Bradley–Terry model can be shown to be equivalent to the logit model [20], given as

$$\text{logit}\,(\pi_{ij}) \equiv \log\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) = \beta_i - \beta_j \tag{8}$$

When there is no order effect, then $1 - \pi_{ij} = \pi_{ji}$. The results of fitting the Bradley–Terry model using the MATLAB generalized linear model to perform the logistic regression are shown in Table 2. Identifiability of the model requires an additional constraint [20]; in this case,

$$\sum_i \hat{\beta}_i = 0$$

was enforced. A test of fit of the model found that it was acceptable ($p > 0.38$). The correlation coefficient between the measured pair probabilities and those predicted by the Bradley–Terry model was $r = 0.93$, indicating that the model predicts the data quite well. An examination of Table 2 shows that the perceived motion strength appears to be an increasing function of both acceleration and jerk.

Because the subject repeatability was quite low, an extended Bradley–Terry model [20] that includes an order effect was fit to the data. In this case, the logit model is

$$\text{logit}\,(\pi^*_{ij}) \equiv \log\left(\frac{\pi^*_{ij}}{1-\pi^*_{ij}}\right) = \gamma + \beta^*_i - \beta^*_j \tag{9}$$

where π^*_{ij} denotes the probability that i is preferred to j when i comes first. When γ is greater than 0, then there is a preference for the first condition. This model was fit to the data and the merit results are shown in Table 2. The order effect γ was found to be −0.74. A likelihood ratio test [20] found that the order parameter was highly significant ($p < 0.0001$). These results indicate a relatively strong preference for the second condition. Thus, when the two motion strengths were similar, the second motion condition was often selected. The correlation coefficient increased to $r = 0.95$ when order was considered, which again shows the importance of the order effect in the subject response data.

Finally, a continuous-logit model was fit to the data, with the independent variables taken to be the jerk, acceleration, and duration of the constant-acceleration phase D, where $D_i = T_{s_i} - 2A_i/J_i$. The model is taken to be

$$\text{logit}(\pi^*_{ij}) \equiv \log\left(\frac{\pi^*_{ij}}{1-\pi^*_{ij}}\right) = \gamma + \beta_A(A_i - A_j) + \beta_J(J_i - J_j) + \beta_D(D_i - D_j) \tag{10}$$

where D is a nonlinear interaction between the jerk and acceleration. The model fit resulted in $\gamma = -0.72$, $\beta_A = 12.3$, $\beta_J = 0.38$, and $\beta_D = 2.39$. An overall test of fit (null hypothesis $\gamma = \beta_A = \beta_J = \beta_D = 0$) found that the model was highly significant ($p < 0.0001$). Subsequent tests of all the parameters found them to be at least marginally significant ($p < 0.0001$, $p < 0.0001$, $p = 0.08$, and $p < 0.0001$, respectively). The correlation coefficient between measured pair probabilities and those predicted by the model is $r = 0.95$, indicating that the model explains much of the data. A test of goodness of fit found that the model fit was acceptable ($p > 0.8$). The pair probabilities predicted by the model are compared with the measured values in Table 3. As can be seen in the table, the four-parameter model predicts the 72-pair probabilities quite well. This indicates that the low consistency was likely due to the similarity of motion conditions and not due to the conditions being compared on multiple scales.

The logit of the paired probabilities is on a linear scale and can be thought of as indicating the motion strength. Although jerk was marginally significant, the effect of jerk on the motion strength was only 1/30th of the effect of acceleration on the motion strength.

The data from this experiment support the hypothesis that in addition to the acceleration, the jerk level of a time history can affect the perceived strength of motion. Thus, scaling down the acceleration but increasing the jerk (which means, of course, that the time history of the motion has been distorted) could lead to an increase in the perceived strength of motion. It should be noted, however, that a very large increase in jerk would be required to overwhelm a modest decrease in acceleration if the duration of the acceleration was kept constant.

To see if the predictions of the model can explain the low scale factors reported in the literature, the model was rearranged so that the acceleration and jerk changes were expressed as dimensionless ratios. This was achieved by scaling the changes in jerk and acceleration by a nominal value, which was the middle of the tested ranges in this case, and so rearranging Eq. (10) yields

$$\text{logit}(\pi^*_{ij}) \equiv \log\left(\frac{\pi^*_{ij}}{1-\pi^*_{ij}}\right) = \gamma + \beta^*_A(A_i - A_j)/A_{\text{nom}} + \beta^*_J(J_i - J_j)/J_{\text{nom}} + \beta_D(D_i - D_j) \tag{11}$$

where $\beta^*_A = 6.15$ and $\beta^*_J = 0.76$. A 400% increase in jerk combined with a 50% decrease in acceleration would therefore not change the

Table 3 Measured and model-predicted pair probabilities

	Second condition								
First condition	A	B	C	D	E	F	G	H	I
A	——	0.36[a] (0.11)[b]	0.05 (0.06)	0.18 (0.19)	0.09 (0.04)	0.05 (0.02)	0.09 (0.09)	0.05 (0.01)	0.09 (0.01)
B	0.45[c] (0.65)	——	0.09 (0.19)	0.55 (0.47)	0.09 (0.15)	0.18 (0.08)	0.09 (0.26)	0.05 (0.05)	0.05 (0.03)
C	0.95 (0.79)	0.45 (0.49)	——	0.73 (0.64)	0.18 (0.27)	0.18 (0.14)	0.36 (0.42)	0.05 (0.10)	0.18 (0.05)
D	0.82 (0.51)	0.09 (0.21)	0.18 (0.12)	——	0.09 (0.09)	0.05 (0.04)	0.18 (0.16)	0.05 (0.03)	0.09 (0.01)
E	0.82 (0.83)	0.73 (0.57)	0.27 (0.39)	0.82 (0.70)	——	0.18 (0.18)	0.64 (0.49)	0.05 (0.13)	0.05 (0.07)
F	0.95 (0.92)	0.55 (0.74)	0.64 (0.59)	0.73 (0.84)	0.45 (0.51)	——	0.73 (0.68)	0.18 (0.24)	0.09 (0.14)
G	0.82 (0.72)	0.36 (0.40)	0.09 (0.25)	0.64 (0.54)	0.36 (0.20)	0.05 (0.10)	——	0.18 (0.07)	0.05 (0.04)
H	0.91 (0.94)	0.95 (0.81)	0.45 (0.68)	0.95 (0.89)	0.55 (0.61)	0.64 (0.42)	0.82 (0.76)	——	0.18 (0.19)
I	0.95 (0.97)	0.82 (0.90)	0.73 (0.81)	0.91 (0.94)	0.64 (0.77)	0.45 (0.60)	0.82 (0.87)	0.45 (0.50)	——

[a] $\pi^*_{A,B}$
[b] Model-predicted values are indicated in parentheses.
[c] $\pi^*_{B,A}$

motion strength. This model seems more likely to work at higher levels of jerk and acceleration than the original model shown in Eq. (10), because it is normalized by the nominal acceleration and jerk.

Examination of this model demonstrates that it can explain the preference for very-small-scale factors when step inputs are used to represent actual aircraft motions, because the jerk is grossly exaggerated. If we assume for the Groen et al. experiment [4] that the jerk increased from 30 (J_{nom}) to 210 $\mathrm{m/s^3}$ (due to use of step input rather than a vehicle model), then if the acceleration decreased from 3.5 (A_{nom}) to 0.9047 $\mathrm{m/s^2}$, the perceived motion strength would be the same (ignoring the effect of the duration). This is in good agreement with the Groen et al. results. It also seems possible that this model could explain the preference for reduced scale factors for more modest increases in jerk, such as those due to the MDA. As shown in Fig. 1, for a typical flight maneuver with nominal MDA parameters, the jerk can be exaggerated by as much as 300%, which, according to Eq. (11), would be require a 38% decrease in acceleration (scale factor of 0.62) to maintain the same motion strength (assuming that the duration of the acceleration does not change). For step-type inputs when no tilt-rate limiting occurs, the MDA will exaggerate the jerk by less than 25%, which would require very little change in the acceleration to maintain a constant motion strength. So in this last case, it seems there must be other factors contributing to the exaggerated perception of motion strength.

Caution must be exercised when extending these results to these more general simulator results. The fit was done using very small accelerations with reasonably large durations and relatively small jerks, whereas typical flight simulator motions have much larger accelerations and jerks, and so the fit may not apply. In addition, the durations of the accelerations in the simulator compared with the aircraft are difficult to access. As seen in Fig. 1, for step-type inputs, the specific force in the simulator will initially match the (scaled) input, but will then droop for some time before the tilt-coordination builds up the specific force to the correct steady-state value. It seems likely that the droop will lead to a reduction in motion strength, but further study is required to confirm this.

V. Conclusions

A paired-comparison experiment was run to test the hypothesis that the perceived strength of motion is a function of both the jerk and acceleration of the motion. Based on a logit model, it was found that both jerk and acceleration contributed positively to the perception of motion strength. The duration of a constant-acceleration phase was also found to have a large effect on the perceived motion strength, with strength increasing with duration. In addition, the data also showed a relatively large effect of order, with the second condition in the pair being preferred.

The results indicate that reproducing motions in the simulator by scaling down acceleration but increasing jerk could create a perception of motion that is too strong. The results of this experiment provide a plausible explanation for why simulator MDA scale factors of less than 1 can still lead to motion that feels too strong, particularly if the vehicle jerk is significantly exaggerated or if tilt-rate limiters are used in the motion drive algorithm. When assessing simulator fidelity, simulation engineers should therefore compare both the jerk and the acceleration time histories of the simulated and actual vehicle.

Because the experiment did not exactly replicate the motions that are typical of the MDA response to step-type inputs from simulated vehicles, the results do not conclusively prove that jerk is a significant contributing factor to preference for relatively small motion scale factors. Further studies should focus on extending these results to larger accelerations and jerks. In addition, an additional study in which the acceleration duration is held constant should be carried out to further refine the effect of jerk on the perceived motion strength.

Acknowledgments

This work was supported by a Natural Sciences and Engineering Research Council (NSERC Canada) Discovery grant and by MTS Systems Corporation, Eden Prairie, Minnesota.

References

[1] Young, L., "Some Effects of Motion Cues on Manual Tracking," *Journal of Spacecraft and Rockets*, Vol. 4, No. 10, 1967, pp. 1300–1303.

[2] Shirley, R. S., and Young, L. R., "Motion Cues in Man-Vehicle Control," *IEEE Transactions on Man–Machine Systems*, Vol. 9, No. 4, 1968, pp. 121–128.
doi:10.1109/TMMS.1968.300016

[3] Schroeder, J. A., "Evaluation of Simulation Motion Fidelity Criteria in the Vertical and Directional Axes," *Journal of the American Helicopter Society*, Vol. 41, No. 2, Apr. 1996, pp. 44–52.

[4] Groen, E. L., Valenti Clari, M. S. V., and Hosman, R. J. A. W., "Evaluation of Perceived Motion During a Simulated Takeoff Run," *Journal of Aircraft*, Vol. 38, No. 4, July–Aug. 2001, pp. 600–606.

[5] Kuge, N., Kubota, M., and Itoh, K., "A Study on a Motion Algorithm Based on a Driver-Centered Approach," *Driving Simulation Conference Europe (DSC2002)* [CD-ROM], Lavoisier, Paris, Sept. 2002.

[6] Kemeny, A., "Multi Sensory Integration and Perceptive Immersion in Driving Simulator Experiments," *Driving Simulation Conference Europe (DSC2004)* [CD-ROM], Lavoisier, Paris, Sept. 2004.

[7] Breidenbach, W. T. C., "The Use of Driving Simulators in Simulating the Off-Road Drive of Wheeled Vehicles," *Driving Simulation Conference Europe (DSC2004)* [CD-ROM], Lavoisier, Paris, Sept. 2004.

[8] Hosman, R. J. A. W., "Pilots Perception and Control of Aircraft Motions," Ph.D. Dissertation, Delft Univ. of Technology, Delft, The Netherlands, Nov. 1996.

[9] Reid, L. D., and Nahon, M., "Flight Simulation Motion-Base Drive Algorithms, Part 1: Developing and Testing the Equations," Univ. of Toronto, Inst. for Aerospace Studies, Rept. 296, 1985.

[10] Reid, L. D., and Nahon, M., "Flight Simulation Motion-Base Drive Algorithms, Part 2: Selecting the System Parameters," Univ. of Toronto, Inst. for Aerospace Studies, Rept. 307, 1986.

[11] Grant, P. R., and Reid, L. D., "Motion Washout Filter Tuning: Rules and Requirements," *Journal of Aircraft*, Vol. 34, No. 2, Mar.–Apr. 1997, pp. 145–151.

[12] Hosman, R. J. A. W., and van der Vaart, J. C., "Vestibular Models and Thresholds of Motion Perception: Results of Tests in a Flight Simulator," Delft Univ. of Technology, Rept. LR-265, Delft, The Netherlands, Apr. 1978.

[13] Heerspink, H., Berkouwer, W., Stroosma, O., Paassen, M. V., Mulder, M., and Mulder, J., "Evaluation of Vestibular Thresholds for Motion Detection in the Simona Research Simulator," AIAA Modeling and Simulation Technologies Conference, San Francisco, AIAA Paper 2005-6502, Aug. 2005.

[14] Mah, R., Young, L. R., Steele, C., and Schubert, E., "Threshold Perception of Whole-Body Motion to Linear Sinusoidal Stimulation," AIAA Flight Simulation Technologies Conference and Exhibit, Boston, MA, AIAA Paper 89-3272, Aug. 1989.

[15] Greig, G. L., "Masking of Motion Cues by Random Motion: Comparison of Human Performance with a Signal Detection Model," Univ. of Toronto, Inst. for Aerospace Studies, Rept. 313, 1988.

[16] Huang, Q., and Wang, H., "Fundamental Study of Jerk: Evaluation of Shift Quality and Ride Comfort," *Proceedings of the 2004 SAE Automotive Dynamics, Stability and Control Conference*, Society of Automotive Engineers, Warrendale, PA, May 2004, pp. 93–98.

[17] Duncan, J., and Wegscheid, E., "Determinants of Off-Road Vehicle Transmission 'Shift' Quality," *Applied ergonomics*, Vol. 16, No. 3, 1985, pp. 173–178.
doi:10.1016/0003-6870(85)90004-3

[18] David, H. A., *The Method of Paired Comparisons*, Oxford Univ. Press, New York, 1988.

[19] Grant, P. R., "Motion Characteristics of the UTIAS Flight Research Simulator Motion Base," Univ. of Toronto, Inst. for Aerospace Studies, TN 261, July 1986.

[20] Agresti, A., *Categorical Data Analysis*, Wiley, New York, 2002.